# CHAPTER 2

# ARRAYS AND STRUCTURES



## Arrays

Array: a set of index and value

data structure
For each index, there is a value associated with that index.

representation (possible)
implemented by using consecutive memory.



**Structure** *Array* is
**objects:** A set of pairs <*index, value*> where for each value of *index* there is a value from the set *item*. *Index* is a finite ordered set of one or more dimensions, for example, {0, … , n-1} for one dimension, {(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,1),(2,2)} for two dimensions, etc.
**Functions:**
for all A ∈ Array, *i* ∈ *index*, x ∈ *item, j, size* ∈ integer
Array Create(j, list) ::= **return** an array of *j* dimensions where list is a j-tuple whose *i*th element is the size of the *i*th dimension. *Items* are undefined.
*Item* Retrieve(A, *i*) ::= **if** (*i* ∈ *index*) **return** the item associated with index value *i* in array A
**else return** error
*Array Store*(A, *i, x*) ::= **if** (*i* in *index*)
**return** an array that is identical to array A except the new pair <*i, x*> has been inserted **else return** error
**end** array

*__Structure 2.1:__ Abstract Data Type *Array* ( p.50)



## Arrays in C

int list[5], *plist[5];

list[5]: five integers
list[0], list[1], list[2], list[3], list[4]
*plist[5]: five pointers to integers
plist[0], plist[1], plist[2], plist[3], plist[4]

**implementation of 1-D array**

| | |
|---|---|
| list[0] | base address = $\alpha$ |
| list[1] | $\alpha$ + sizeof(int) |
| list[2] | $\alpha$ + 2*sizeof(int) |
| list[3] | $\alpha$ + 3*sizeof(int) |
| list[4] | $\alpha$ + 4*size(int) |



## Arrays in C (*Continued*)

Compare int *list1 and int list2[5] in C.

Same: list1 and list2 are pointers.
Difference: list2 reserves five locations.

Notations:
list2 - a pointer to list2[0]
(list2 + i) - a pointer to list2[i] (&list2[i])
*(list2 + i) - list2[i]



## Example: 1-dimension array addressing

int one[] = {0, 1, 2, 3, 4};
Goal: print out address and value

```
void print1(int *ptr, int rows)
{
/* print out a one-dimensional array using a pointer */
    int i;
    printf("Address Contents\n");
    for (i=0; i < rows; i++)
        printf("%8u%5d\n", ptr+i, *(ptr+i));
    printf("\n");
}
```

call print1(&one[0], 5)

| Address | Contents |
|---|---|
| 1228 | 0 |
| 1230 | 1 |
| 1232 | 2 |
| 1234 | 3 |
| 1236 | 4 |

*__Figure 2.1:__ One- dimensional array addressing (p.53)



# Structures (records)

```
struct {
        char name[10];
        int age;
        float salary;
        } person;

strcpy(person.name, “james”);
person.age=10;
person.salary=35000;
```



# Create structure data type

```
typedef struct human_being {
        char name[10];
        int age;
        float salary;
        };

or

typedef struct {
        char name[10];
        int age;
        float salary
        } human_being;

human_being person1, person2;
```



# Unions

Similar to struct, but only one field is active.
Example: Add fields for male and female.

```
typedef struct sex_type {
        enum tag_field {female, male} sex;
        union {
                int children;
                int beard;
                } u;
        };
typedef struct human_being {
        char name[10];
        int age;
        float salary;
        date dob;
        sex_type sex_info;
        }
```

```
human_being person1, person2;
person1.sex_info.sex=male;
person1.sex_info.u.beard=FALSE;
```



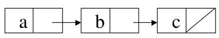
# Self-Referential Structures

One or more of its components is a pointer to itself.

```
typedef struct list {
        char data;
        list *link;
        }
```

Construct a list with three nodes
item1.link=&item2;
item2.link=&item3;
malloc: obtain a node

```
list item1, item2, item3;
item1.data=‘a’;
item2.data=‘b’;
item3.data=‘c’;
item1.link=item2.link=item3.link=NULL;
```






# Ordered List Examples

ordered (linear) list: (item1, item2, item3, …, item*n*)

- (MONDAY, TUEDSAY, WEDNESDAY, THURSDAY, FRIDAY, SATURDAYY, SUNDAY)
- (2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, Ace)
- (1941, 1942, 1943, 1944, 1945)
- ($a_1$, $a_2$, $a_3$, …, $a_{n-1}$, $a_n$)

# Operations on Ordered List

- Find the length, n , of the list.
- Read the items from left to right (or right to left).
- Retrieve the i'th element.
- Store a new value into the i'th position.
- Insert a new element at the position i , causing elements numbered i, i+1, …, n to become numbered i+1, i+2, …, n+1
- Delete the element at position i , causing elements numbered i+1, …, n to become numbered i, i+1, …, n-1 array (sequential mapping)? (1)~(4) O (5)~(6) X



Polynomials A(X)=3X$^{20}$+2X$^{5}$+4, B(X)=X$^{4}$+10X$^{3}$+3X$^{2}$+1

**Structure** *Polynomial* is

**objects**: $p(x) = a_1x^{e_1} + \ldots + a_nx^{e_n}$ ; a set of ordered pairs of <$e_i$,$a_i$> where $a_i$ in *Coefficients* and $e_i$ in *Exponents*, $e_i$ are integers >= 0

**functions:**

for all *poly, poly1, poly2* ∈ *Polynomial*, *coef* ∈*Coefficients*, *expon* ∈*Exponents*

*Polynomial* Zero( ) ::= **return** the polynomial, *p(x)* = 0

*Boolean* IsZero(*poly*) ::= **if** (*poly*) **return** *FALSE* **else return** *TRUE*

*Coefficient* Coef(*poly, expon*) ::= **if** (*expon* ∈ *poly*) **return** its coefficient **else return** Zero

*Exponent* Lead_Exp(*poly*) ::= **return** the largest exponent in *poly*

*Polynomial* Attach(*poly,coef, expon*) ::= **if** (*expon* ∈ *poly*) **return** error **else return** the polynomial poly with the term <*coef, expon*> inserted



*Polynomial* Remove(*poly, expon*) ::= **if** (*expon* ∈ *poly*) **return** the polynomial *poly* with the term whose exponent is *expon deleted* **else return** error

*Polynomial* SingleMult(*poly, coef, expon*) ::= **return** the polynomial *poly* • *coef* • $x^{expon}$

*Polynomial* Add(*poly1, poly2*) ::= **return** the polynomial *poly*1 +*poly*2

*Polynomial* Mult(*poly1, poly2*) ::= **return** the polynomial *poly*1 • *poly*2

**End** *Polynomial*

***Structure 2.2:**Abstract data type *Polynomial* (p.61)



## Polynomial Addition

data structure 1:

```
#define MAX_DEGREE 101
typedef struct {
        int degree;
        float coef[MAX_DEGREE];
        } polynomial;
```

```
/* d =a + b, where a, b, and d are polynomials */
d = Zero( )
while (! IsZero(a) && ! IsZero(b)) do {
  switch COMPARE (Lead_Exp(a), Lead_Exp(b))  {
    case -1: d =
      Attach(d, Coef (b, Lead_Exp(b)), Lead_Exp(b));
      b = Remove(b, Lead_Exp(b));
      break;
    case  0: sum = Coef (a, Lead_Exp (a)) + Coef ( b, Lead_Exp(b));
      if (sum) {
        Attach (d, sum, Lead_Exp(a));
        a = Remove(a , Lead_Exp(a));
        b = Remove(b , Lead_Exp(b));
        }
      break;
```



```
case 1: d =
       Attach(d, Coef (a, Lead_Exp(a)), Lead_Exp(a));
       a = Remove(a, Lead_Exp(a));
     }
  }
insert any remaining terms of a or b into d
```

advantage: easy implementation

disadvantage: waste space when sparse

***Program 2.4 :**Initial version of *padd* function(p.62)



Data structure 2: use one global array to store all polynomials

A(X)=2X$^{1000}$+1

B(X)=X$^{4}$+10X$^{3}$+3X$^{2}$+1

***Figure 2.2:** Array representation of two polynomials (p.63)

starta (0) finisha (1) startb (2) finishb (5) avail (6)

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| *coef* | 2 | 1 | 1 | 10 | 3 | 1 | |
| *exp* | 1000 | 0 | 4 | 3 | 2 | 0 | |

| specification poly | representation <start, finish> |
|---|---|
| A | <0,1> |
| B | <2,5> |

storage requirements: start, finish, 2*(finish-start+1)
nonparse: twice as much as (1)
when all the items are nonzero

```
MAX_TERMS 100 /* size of terms array */
typedef struct {
        float coef;
        int expon;
        } polynomial;
polynomial terms[MAX_TERMS];
int avail = 0;
```

*(p.62)



## Add two polynomials: D = A + B

```
void padd (int starta, int finisha, int startb, int finishb,
                                    int * startd, int *finishd)
{
/* add A(x) and B(x) to obtain D(x) */
   float coefficient;
  *startd = avail;
  while (starta <= finisha && startb <= finishb)
   switch (COMPARE(terms[starta].expon,
                   terms[startb].expon)) {
    case -1: /* a expon < b expon */
         attach(terms[startb].coef, terms[startb].expon);
         startb++
         break;
```



```
case  0: /* equal exponents */
          coefficient = terms[starta].coef +
                        terms[startb].coef;
          if (coefficient)
             attach (coefficient, terms[starta].expon);
          starta++;
          startb++;
          break;
case 1: /* a expon > b expon */
      attach(terms[starta].coef, terms[starta].expon);
      starta++;
}
```



```
/* add in remaining terms of  A(x) */
for( ; starta <= finisha; starta++)
    attach(terms[starta].coef, terms[starta].expon);
/* add in remaining terms of B(x) */
for( ; startb <= finishb; startb++)
    attach(terms[startb].coef, terms[startb].expon);
*finishd =avail -1;
}
```

Analysis: O(n+m)
where n (m) is the number of nonzeros in A(B).

***Program 2.5:** Function to add two polynomial (p.64)



```
void attach(float coefficient, int exponent)
{
/* add a new term to the polynomial */
   if (avail >= MAX_TERMS) {
     fprintf(stderr, "Too many terms in the polynomial\n");
     exit(1);
   }
   terms[avail].coef  = coefficient;
   terms[avail++].expon = exponent;
}
```

***Program 2.6:**Function to add anew term (p.65)

Problem: Compaction is required
when polynomials that are no longer needed.
(data movement takes time.)



## Sparse Matrix

$$\begin{array}{c} \\ \text{row 1} \\ \text{row 2} \\ \text{row 3} \\ \text{row 4} \\ \text{row 5} \end{array} \begin{array}{c} \begin{array}{ccc} \text{col 1} & \text{col 2} & \text{col 3} \end{array} \\ \begin{bmatrix} -27 & 3 & 4 \\ 6 & 82 & -2 \\ 109 & -64 & 11 \\ 12 & 8 & 9 \\ 48 & 27 & 47 \end{bmatrix} \end{array} \; 5*3$$

$$\begin{array}{c} \\ \text{row0} \\ \text{row1} \\ \text{row2} \\ \text{row3} \\ \text{row4} \\ \text{row5} \end{array} \begin{array}{c} \begin{array}{cccccc} \text{col1} & \text{col2} & \text{col3} & \text{col4} & \text{col5} & \text{col6} \end{array} \\ \begin{bmatrix} 15 & 0 & 0 & 22 & 0 & -15 \\ 0 & 11 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 91 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 28 & 0 & 0 & 0 \end{bmatrix} \end{array} \; 6*6$$

(a) 15/15 (b) 8/36

***Figure 2.3:Two matrices**

sparse matrix
data structure?

SPARSE MATRIX ABSTRACT DATA TYPE

**Structure** *Sparse_Matrix* is

**objects:** a set of triples, <*row, column, value*>, where *row* and *column* are integers and form a unique combination, and *value* comes from the set *item.*

**functions**:

for all *a, b* ∈ *Sparse_Matrix*, *x* ∈ *item, i, j, max_col, max_row* ∈ *index*

*Sparse_Marix* Create(*max_row, max_col*) ::=
**return** a *Sparse_matrix* that can hold up to *max_items* = *max _row* × *max_col* and whose maximum row size is *max_row* and whose maximum column size is *max_col.*



*Sparse_Matrix* Transpose(*a*) ::=
**return** the matrix produced by interchanging the row and column value of every triple.

*Sparse_Matrix* Add(*a, b*) ::=
**if** the dimensions of a and b are the same
**return** the matrix produced by adding corresponding items, namely those with identical *row* and *column* values.
**else return** error

*Sparse_Matrix* Multiply(*a, b*) ::=
**if** number of columns in a equals number of rows in **b**
**return** the matrix *d* produced by multiplying a by *b* according to the formula: *d* [*i*] [*j*] = Σ(a[i][k]•b[k][j]) where *d (i, j)* is the *(i,j)*th element
**else return** error.



*** Structure 2.3:** Abstract data type Sparse-Matrix (p.68)

(1) Represented by a two-dimensional array.
Sparse matrix wastes space.

(2) Each element is characterized by <row, col, value>.

# of rows (columns)
# of nonzero terms

| | row | col | value | | | row | col | value |
|---|---|---|---|---|---|---|---|---|
| a[0] | 6 | 6 | 8 | | b[0] | 6 | 6 | 8 |
| [1] | 0 | 0 | 15 | | [1] | 0 | 0 | 15 |
| [2] | 0 | 3 | 22 | | [2] | 0 | 4 | 91 |
| [3] | 0 | 5 | -15 | | [3] | 1 | 1 | 11 |
| [4] | 1 | 1 | 11 | transpose → | [4] | 2 | 1 | 3 |
| [5] | 1 | 2 | 3 | | [5] | 2 | 5 | 28 |
| [6] | 2 | 3 | -6 | | [6] | 3 | 0 | 22 |
| [7] | 4 | 0 | 91 | | [7] | 3 | 2 | -6 |
| [8] | 5 | 2 | 28 | | [8] | 5 | 0 | -15 |

(a) (b)

row, column in ascending order

***Figure 2.4:**Sparse matrix and its transpose stored as triples (p.69)



```
Sparse_matrix Create(max_row, max_col) ::=

#define MAX_TERMS 101 /* maximum number of terms +1*/
   typedef struct {
           int col;
           int row;
           int value;
           } term;
   term a[MAX_TERMS]
```

# of rows (columns)
# of nonzero terms

* (P.69)



## Transpose a Matrix

(1) for each row i
take element <i, j, value> and store it in element <j, i, value> of the transpose.

difficulty: where to put <j, i, value>
(0, 0, 15) ====> (0, 0, 15)
(0, 3, 22) ====> (3, 0, 22)
(0, 5, -15) ====> (5, 0, -15)
(1, 1, 11) ====> (1, 1, 11)
Move elements down very often.

(2) For all elements in column j,
place element <i, j, value> in element <j, i, value>



```
void transpose (term a[], term b[])
/* b is set to the transpose of a */
{
  int n, i, j, currentb;
  n = a[0].value;  /* total number of elements */
  b[0].row = a[0].col;  /* rows in b = columns in a */
  b[0].col = a[0].row;  /*columns in b = rows in a */
  b[0].value = n;
  if (n > 0) {              /*non zero matrix */
    currentb = 1;
    for (i = 0; i < a[0].col; i++)
    /* transpose by columns in a */
       for( j = 1; j <= n; j++)
       /*  find elements from the current column */
       if (a[j].col == i) {
      /* element is in current column, add it to b */
```

```
columns
    elements
        b[currentb].row = a[j].col;
        b[currentb].col  = a[j].row;
        b[currentb].value = a[j].value;
        currentb++
      }
  }
}
```

*** Program 2.7:** Transpose of a sparse matrix (p.71)

Scan the array "columns" times.
The array has "elements" elements. ==> O(columns*elements)



Discussion: compared with 2-D array representation

O(columns*elements) vs. O(columns*rows)

elements --> columns * rows when nonsparse
O(columns*columns*rows)

Problem: Scan the array "columns" times.

Solution:
Determine the number of elements in each column of the original matrix.
==>
Determine the starting positions of each row in the transpose matrix.



| | | | |
|---|---|---|---|
| a[0] | 6 | 6 | 8 |
| a[1] | 0 | 0 | 15 |
| a[2] | 0 | 3 | 22 |
| a[3] | 0 | 5 | -15 |
| a[4] | 1 | 1 | 11 |
| a[5] | 1 | 2 | 3 |
| a[6] | 2 | 3 | -6 |
| a[7] | 4 | 0 | 91 |
| a[8] | 5 | 2 | 28 |

| | [0] | [1] | [2] | [3] | [4] | [5] |
|---|---|---|---|---|---|---|
| row_terms = | 2 | 1 | 2 | 2 | 0 | 1 |
| starting_pos = | 1 | 3 | 4 | 6 | 8 | 8 |



```
void fast_transpose(term a[ ], term b[ ])
{
/* the transpose of a is placed in b */
  int row_terms[MAX_COL], starting_pos[MAX_COL];
  int i, j, num_cols = a[0].col, num_terms = a[0].value;
  b[0].row = num_cols; b[0].col = a[0].row;
  b[0].value = num_terms;
  if (num_terms > 0){ /*nonzero matrix*/
columns   for (i = 0; i < num_cols; i++)
              row_terms[i] = 0;
elements  for (i = 1; i  <= num_terms; i++)
              row_term [a[i].col]++
    starting_pos[0] = 1;
columns   for (i =1; i < num_cols; i++)
              starting_pos[i]=starting_pos[i-1] +row_terms [i-1];
```



```
          for (i=1; i <= num_terms, i++) {
                j = starting_pos[a[i].col]++;
elements        b[j].row = a[i].col;
                b[j].col = a[i].row;
                b[j].value = a[i].value;
          }
      }
  }
```

***Program 2.8:** Fast transpose of a sparse matrix

Compared with 2-D array representation
O(columns+elements) vs. O(columns*rows)
elements --> columns * rows
O(columns+elements) --> O(columns*rows)

Cost: Additional row_terms and starting_pos arrays are required.
Let the two arrays row_terms and starting_pos be shared.



## Sparse Matrix Multiplication

Definition: $[D]_{m*p}=[A]_{m*n}*[B]_{n*p}$
Procedure: Fix a row of A and find all elements in column j of B for j=0, 1, …, p-1.

Alternative 1. Scan all of B to find all elements in j.
Alternative 2. Compute the transpose of B.
(Put all column elements consecutively)

$$\begin{bmatrix}1 & 0 & 0\\1 & 0 & 0\\1 & 0 & 0\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}=\begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}$$



***Figure 2.5:** Multiplication of two sparse matrices (p.73)

## An Example

$$A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 4 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 0 & 2 \\ -1 & 0 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

| | | |
|---|---|---|
| 2 | 3 | 5 |
| 0 | 0 | 1 |
| 0 | 2 | 2 |
| 1 | 0 | -1 |
| 1 | 1 | 4 |
| 1 | 2 | 6 |

| | | |
|---|---|---|
| 3 | 3 | 4 |
| 0 | 0 | 3 |
| 0 | 2 | 2 |
| 1 | 0 | -1 |
| 2 | 2 | 5 |

$$B^T = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 5 \end{bmatrix}$$

(0,0) (0,2) (1,0) (1,2)

| | | |
|---|---|---|
| 3 | 3 | 4 |
| 0 | 0 | 3 |
| 0 | 1 | -1 |
| 2 | 0 | 2 |
| 2 | 2 | 5 |



## General Case

$d_{ij}=a_{i0}*b_{0j}+a_{i1}*b_{1j}+\ldots+a_{i(n-1)}*b_{(n-1)j}$

a本來依i成群，經轉置後，b也依j成群。

| | | |
|---|---|---|
| a | Sa | |
| b | Sb | |
| c | Sc | |

| | | |
|---|---|---|
| d | Sd | |
| e | Se | |
| f | Sf | |
| g | Sg | |

最多可以產生ad, ae, af, ag,
bd, be, bf, bg,
cd, ce, cf, cg 等entries 。



```
void mmult (term a[ ], term b[ ], term d[ ] )
/* multiply two sparse matrices */
{
   int i, j, column, totalb = b[].value, totald = 0;
   int rows_a = a[0].row, cols_a = a[0].col,
   totala = a[0].value; int cols_b = b[0].col,
   int row_begin = 1, row = a[1].row, sum =0;
   int new_b[MAX_TERMS][3];
   if (cols_a != b[0].row){
         fprintf (stderr, "Incompatible matrices\n");
         exit (1);
}
```



```
fast_transpose(b, new_b);                cols_b + totalb
/* set boundary condition */
a[totala+1].row = rows_a;
new_b[totalb+1].row = cols_b;
new_b[totalb+1].col = 0;
for (i = 1; i <= totala; ) {             at most rows_a times
    column = new_b[1].row;
    for (j = 1; j <= totalb+1;) {
    /* mutiply row of a by column of b */
    if (a[i].row != row)  {
       storesum(d, &totald, row, column, &sum);
       i = row_begin;
       for (; new_b[j].row == column; j++)
         ;
       column =new_b[j].row
    }
```



```
    else switch (COMPARE (a[i].col, new_b[j].col)) {
        case -1: /* go to next term in a */
                 i++; break;
        case 0: /* add terms, go to next term in a and b */
                 sum += (a[i++].value * new_b[j++].value);
                 break;
         case 1: /* advance to next term in b*/
                  j++
       }
    } /* end of for j <= totalb+1 */
    for (; a[i].row == row; i++)
         ;
     row_begin = i; row = a[i].row;
  } /* end of for i <=totala */
  d[0].row = rows_a;
  d[0].col = cols_b; d[0].value = totald;
}
```

***Praogram 2.9:** Sparse matrix multiplication (p.75)



# Analyzing the algorithm

$cols\_b * termsrow_1 + totalb +$
$cols\_b * termsrow_2 + totalb +$
$\ldots +$
$cols\_b * termsrow_p + totalb$
$= cols\_b * (termsrow_1 + termsrow_2 + \ldots + termsrow_p) +$
$rows\_a * totalb$
$= cols\_b * totala + row\_a * totalb$

$O(cols\_b * totala + rows\_a * totalb)$

Compared with matrix multiplication using array

```
for (i =0; i < rows_a; i++)
   for (j=0; j < cols_b; j++) {
      sum =0;
      for (k=0; k < cols_a; k++)
           sum += (a[i][k] *b[k][j]);
      d[i][j] =sum;
   }
```

O(rows_a * cols_a * cols_b) vs.
O(cols_b * total_a + rows_a * total_b)

optimal case: total_a < rows_a * cols_a
total_b < cols_a * cols_b
worse case: total_a --> rows_a * cols_a, or
total_b --> cols_a * cols_b



```
void storesum(term d[ ], int *totald, int row, int column,
                                              int *sum)
{
/* if *sum != 0, then it along with its row and column
   position is stored as the *totald+1 entry in d */
   if (*sum)
      if (*totald < MAX_TERMS) {
         d[++*totald].row = row;
         d[*totald].col = column;
         d[*totald].value = *sum;
      }
     else {
        fprintf(stderr, "Numbers of terms in product
                                      exceed %d\n", MAX_TERMS);
  exit(1);
  }
}
```



**Program 2.10:** storsum function